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DATA REDUCTION AND MULTIPLE COULOMB SCATTERING ERROR ANALYSIS WITHIN SILICON TRACKING DETECTOR SYSTEM

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Particle track reconstruction capabilities of the silicon tracking detector system have been studied. As the multiple Coulomb scattering (MCS) induces unavoidable uncertainties on the coordinate measurement, the corresponding error estimates and the associated correlations have been used to find the best track fit parameters and their errors. Finally it permits one to find the proper particle characteristics, as vertex position and resolution, momentum value, flight direction and the corresponding errors.

The investigation has been performed at the Laboratory of High Energies, JINR.

Редукция данных и анализ ошибок при многократном рассеянии в кремниевых трековых детекторах

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Изучена возможность реконструкции треков в системе кремниевых детекторов. Выбор параметров трека и оценка их ошибок проводились с учетом неустранимых неопределенностей в координатах, к которым приводит многократное кулоновское рассеяние. В итоге это позволило найти правильные параметры рожденной частицы, такие как положение и точность нахождения вершины, величину импульса, направление вылета и соответствующие ошибки.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

1. Introduction

Design and preparation of any components of a detector system must take care of characteristics and detection performances (efficiency, acceptance, position or energy resolution) necessary for a specific process study. The silicon tracking detector system, in our case, must furnish the best information about the coordinate track intercept of the incident particle on

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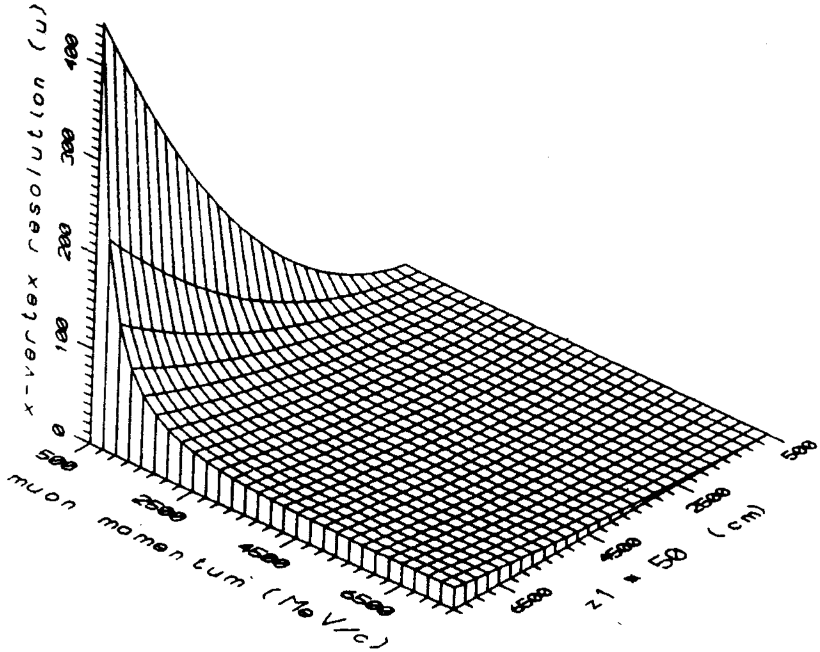

Fig.4. The x-vertex position error (reconstructed vertex resolution deterioration) due to multiple Coulomb scattering as a function of muon momentum and distance from the interaction point to first detector layer (z_1)

Table 4. Multiple Coulomb Scattering limitation of the muon track reconstructed vertex resolution (microns). Table shows its dependence on z_1-position of the first detector layer (cm) and on the muon momentum (MeV/c)

z_1 (cm) / p (MeV)	10	30	50	70	90	110	130	150
500	2.070	18.041	49.831	97.272	160.847	239.639	334.671	446.144
1000	0.681	5.701	15.617	30.420	50.122	74.732	104.258	138.614
1500	0.390	3.145	8.555	16.620	27.333	40.706	56.719	75.436
2000	0.282	2.200	5.942	11.506	18.897	28.114	39.174	52.011
2500	0.230	1.750	4.699	9.080	14.890	22.124	30.795	40.901
3000	0.202	1.502	4.014	7.742	12.680	18.834	26.195	34.773
3500	0.185	1.351	3.597	6.926	11.335	16.828	23.403	31.055
4000	0.173	1.252	3.326	6.396	10.460	15.520	21.574	28.628
4500	0.165	1.184	3.139	6.031	9.856	14.620	20.319	26.955
5000	0.160	1.136	3.005	5.767	9.423	13.973	19.416	25.753

Table 2. Kinematical parameters determination and their associated errors by track reconstruction in the silicon tracking system. Data are based on the muon Monte-Carlo generated events under theta = 15 degree, for a lot of discrete moments

Generated moment (MeV/c)	Reconstructed	
	moment (MeV/c)	theta (deg)
500	500.53 ± 0.44	14.98 ± 0.19
1000	1001.67 ± 1.98	15.02 ± 0.42
1500	1497.13 ± 7.10	15.01 ± 1.01
2000	1997.32 ± 8.39	14.99 ± 0.90
2500	2501.33 ± 15.71	14.99 ± 1.34
3000	3002.76 ± 22.23	14.99 ± 1.58
3500	3497.56 ± 18.74	15.00 ± 1.15
4000	3998.92 ± 23.02	15.00 ± 1.23
4500	4502.71 ± 65.78	14.99 ± 3.13
5000	5003.13 ± 48.06	15.00 ± 2.05
5500	5500.00 ± 117.62	15.02 ± 4.57
6000	6002.02 ± 81.65	15.00 ± 2.91
6500	6498.90 ± 112.23	14.99 ± 3.69
7000	7002.47 ± 141.03	15.00 ± 4.31
7500	7500.07 ± 136.04	15.00 ± 3.88

Table 3. Kinematical parameters determination and their associated errors by track reconstruction in the silicon tracking system. Data are based on the muon Monte-Carlo generated events with Momenta = 500 MeV, for a lot of discrete polar angles

Generated theta (deg)	Reconstructed	
	theta (deg)	moment (MeV/c)
5	5.04 ± 0.28	500.42 ± 0.22
10	10.00 ± 0.09	499.88 ± 0.14
15	14.99 ± 0.22	501.00 ± 0.52
20	19.95 ± 0.55	503.57 ± 1.74
25	24.89 ± 0.16	501.17 ± 0.63
30	30.10 ± 0.49	494.92 ± 2.43
35	35.03 ± 0.28	496.65 ± 1.69
40	40.07 ± 0.31	495.55 ± 2.26
45	45.10 ± 0.12	501.54 ± 1.07

$$\chi^2 = (X - HA_x)^T V^{-1} (X - HA_x), \tag{16}$$

where

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix}; \quad H = \begin{pmatrix} 1 & t_1 \\ 1 & t_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & t_n \end{pmatrix}; \quad A_x = \begin{pmatrix} x_0 \\ v_{0x} \end{pmatrix}$$

the V matrix is given by relation (12) and (14).
Least squares criterion imposes

$$\frac{\partial \chi^2}{\partial A_x} = 0 \quad \text{or} \quad H^T V^{-1} (X - HA_x) = 0.$$

By solving the linear system relative to A_x we can get the fit parameters

$$A_x = (H^T V^{-1} H)^{-1} (H^T V^{-1} X) \tag{17}$$

and the errors of these parameters

$$E_{Ax} \equiv \langle \delta A_x \delta A_x^T \rangle = (H^T V^{-1} H)^{-1}. \tag{18}$$

The same procedure we applied to y and z coordinate, as to find the fit parameters and errors, even for z_i which has no uncertainty, (they are detector position coordinates).

Ones finding the fit parameters (x_0, v_{0x}; y_0, v_{0y}; z_0, v_{0z}) and their errors (σ_{x0}, σ_{v0x}; σ_{y0}, σ_{v0y}; σ_{z0}, σ_{v0z}) it is possible to express the particle kinematical parameters: p-momentum and θ-direction, along with the corresponding errors. In Table 2 and Table 3 there are results on the reconstructed p-momentum and θ-direction values in comparison with the generated ones.

The vertex position error, due to MCS in the detector material, has been also calculated and depends, of course, on particle momentum. If we want to reduce this error it is necessary to bring closer the detector tracking system relative to interaction point. The vertex position error (resolution) dependence on particle (muon) momentum and z_1 distance to first detector layer is shown in Fig.4. The result is useful in detector system design to find an optimal configuration in preparing the experimental work and also to have a choice for vertex resolution as a compromise between the best possible values in the proximity of the interaction point and the worse ones at the radiation harmless distance.

Having in view also the uncorrelated position measurement intrinsic errors σ_0, along with the MCS correlated ones, in the position error matrix V, they will be added in squares (σ_0^2) into the diagonal terms.

For example, for a 500 MeV/c muon track detection by our system configuration ($z_1 = 130$ cm, $z_2 = 131.5$ cm, $z_3 = 133$ cm, $z_4 = 134.5$ cm and $\sigma_0 = 10\,\mu$) the matrix (V_{ij}) and (ρ_{ij}) are

$$V = \begin{pmatrix} 0.1000E{-5} & 0.0000E{+0} & 0.0000E{+0} & 0.0000E{+0} \\ 0.0000E{+0} & 0.4406E{-5} & 0.6813E{-5} & 0.1022E{-4} \\ 0.0000E{+0} & 0.6813E{-5} & 0.1803E{-4} & 0.2725E{-4} \\ 0.0000E{+0} & 0.1022E{-4} & 0.2725E{-4} & 0.4869E{-4} \end{pmatrix},$$

$$\rho = \begin{pmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.7643 & 0.6977 \\ 0.0000 & 0.7643 & 1.0000 & 0.9197 \\ 0.0000 & 0.6977 & 0.9197 & 1.0000 \end{pmatrix}. \qquad (14)$$

As long as the V_{ij} matrix elements depend both on kinematical characteristics of the detected particles and on the tracking detector system configuration, the ρ_{ij} matrix elements are independent of particle characteristics, and are defined just by system configuration.

In the following we will use these matrices in the track reconstruction by a least squares fit procedure.

5. Track Reconstruction Parameters and Their Errors

In the absence of the magnetic field, the unscattered track is a straight line. The independent description of x and y MCS data permits a separate least squares fit to these data by a linear relationship [6]

$$\begin{cases} x = x_0 + v_{0x}t \\ y = y_0 + v_{0y}t \\ z = z_0 + v_{0z}t\,. \end{cases} \qquad (15)$$

With the coordinate and error data $(x_i \pm \sigma_{xi})$, $(y_i \pm \sigma_{yi})$, z_i along with the corresponding correlation matrix ρ_{ij} $(V_{ij} = \rho_{ij}\sigma_{xi}\sigma_{xj})$ as input data, it is possible to express the χ^2 in matrix form, for every coordinate data set. For x-data set it will be

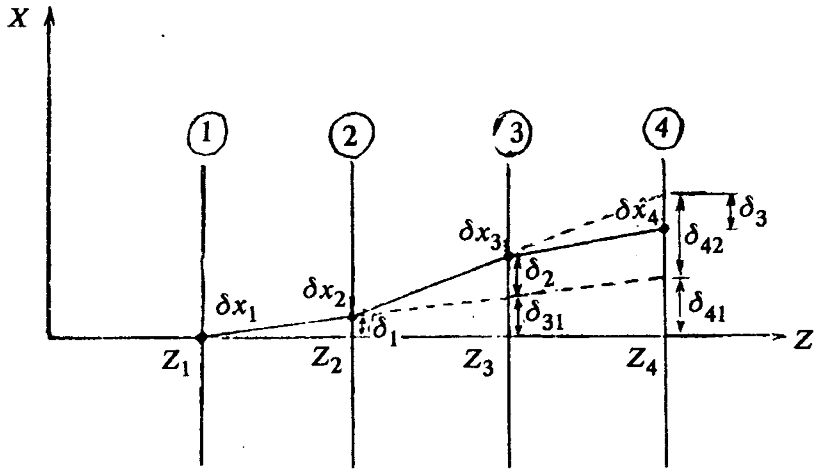

Fig.3. Multiple Coulomb scattering error correlations

Because they are statistical variables it is necessary to find their mean value, and to express it by the independent scattering deviations δ_k, unaffected by earlier scatterings

$$\delta_k \equiv \sqrt{\langle \delta_k^2 \rangle} = \theta_0 (z_{k-1} - z_k). \tag{10}$$

Now let's try to express the δx_i deviations by the independent δ_k ones (10), for which $\langle \delta_k \rangle = 0$,

$$\delta x_i = \sum_{j=1}^{i-1} (z_{j+1} - z_j) \sum_{k=1}^{j} \frac{\delta_k}{(z_{k-1} - z_k)}. \tag{11}$$

Now the position error (covariance) matrix (V_{ij}), defined as the statistical mean of the pair deviation products $\langle \delta x_i \delta x_j \rangle$ for all possible detection layers is

$$V_{ij} \equiv \langle \delta x_i \delta x_j \rangle =$$

$$= \theta_0^2 \, [(z_i - z_1)(z_j - z_1) + (z_i - z_2)(z_j - z_2) + \ldots + (z_i - z_{i-1})(z_j - z_{i-1})]$$

$$\text{for } \; i \leq j = 1, 2, \ldots, n. \tag{12}$$

The (V_{ij}) matrix is symmetric. The error correlation matrix is immediately

$$\rho_{ij} = \frac{\langle \delta x_i \delta x_j \rangle}{\sqrt{\langle \delta x_i^2 \rangle \langle \delta x_j^2 \rangle}}. \tag{13}$$

In Monte-Carlo simulated particle transport the particle position uncertainty on every detector layer has been measured as the variance or the mean square deviation of the scattered track incidence points x_m about the unscattered one x_0

$$\sigma_x^2 = \sum_{m=1}^{N} \frac{(x_m - x_0)^2}{N} \tag{6}$$

and similar for σ_y^2 variance, for every detector layer. The x_m (and y_m) are the coordinates of the scattered track incidence points on detector layer, and N is the total number of generated events.

The distribution widths σ_{xi} of the muon track x-coordinate points on detector layer $i = 2, 3, 4$ and for momentum values from 500 to 7500 MeV/c, are presented in Table 1. There is also the analytical estimation of the same widths of the MCS incidence points distribution on each of the i-detector layer, according to (4)

$$\sigma_{xi}^2 = \frac{i(i-1)(2i-1)}{6} (l\theta_0)^2, \tag{7}$$

where l is the distance between detector layers ($l = 1.5$ cm) and θ_0 is the plane r.m.s. scattering angle (1).

4. Position Error Correlations

MCS produces errors correlated from one layer to the next. Clearly the scattering in layer 1 produces correlated position errors in layer 2, 3 and so on (see Fig.3). The proper error matrix is non-diagonal, and it must be find out.

Let's denote δx_i the track deviation x-coordinate point on the i-th layer, with respect to the initial incident direction on the detector system, then

$$\begin{cases} \delta x_1 = 0 \\ \delta x_2 = \delta_1 \\ \delta x_3 = \delta_{31} + \delta_2 \\ \delta x_4 = \delta_{41} + \delta_{42} + \delta_3, \end{cases} \tag{8}$$

where the individual contributions due to preceding scatterings are

$$\delta_{31} = \delta_1 \frac{z_3 - z_1}{z_2 - z_1}; \qquad \delta_{41} = \delta_1 \frac{z_4 - z_1}{z_2 - z_1}; \qquad \delta_{42} = \delta_2 \frac{z_4 - z_2}{z_3 - z_2}. \tag{9}$$

Table 1. Muon x-distribution width on detector layers of the silicon tracking system due to multiple Coulomb scattering

MONTE-CARLO SIMULATION DATA

P (MeV/c)	σ_{x2} (μm)	σ_{x3} (μm)	σ_{x4} (μm)
500	18.57394	41.28271	68.6337
1000	9.13934	20.25132	33.85192
1500	6.152002	13.61125	22.69169
2000	4.535603	10.09581	16.89462
2500	3.667112	8.081978	13.47147
3000	3.062511	6.778929	11.27552
3500	2.604058	5.776673	9.611728
4000	2.277256	5.074125	8.466508
4500	2.019666	4.489617	7.534582
5000	1.838633	4.082824	6.820512
5500	1.669245	3.721596	6.214138
6000	1.515782	3.377297	5.620477
6500	1.419424	3.130757	5.20202
7000	1.299295	2.884509	4.815011
7500	1.208961	2.677731	4.481739

ANALYTICAL CALCULATION DATA

P (MeV/c)	σ_{x2} (μm)	σ_{x3} (μm)	σ_{x4} (μm)
500	18.45638	41.27002	68.05807
1000	9.079048	20.30138	33.9707
1500	6.034108	13.49268	22.57757
2000	4.520692	10.10857	16.91488
2500	3.61474	8.082806	13.52512
3000	3.011463	6.733836	11.26786
3500	2.58083	5.770912	9.656585
4000	2.257985	5.049009	8.448607
4500	2.006951	4.487679	7.509323
5000	1.806161	4.0387	6.758038
5500	1.641901	3.671403	6.143432
6000	1.505032	3.365353	5.631313
6500	1.389228	3.106409	5.198016
7000	1.289974	2.88447	4.826642
7500	1.203958	2.692132	4.504799

3. The Monte-Carlo Particle Scattering Description

The change of track parameters is usually described by the $\delta\theta_{\text{plane}, x}$ and $\delta\theta_{\text{plane}, y}$ (or $\delta\theta_{\text{space}}$ and $\delta\varphi$) angles and a corresponding displacement δx_{plane} and δy_{plane} in the position.

Following the stochastic nature of the MCS, we use the Monte-Carlo study by generating the joint (δx_{plane}, $\delta\theta_{\text{plane}, x}$) distribution with independent Gaussian random variables (w_1, x_2) [4]

$$\begin{cases} \delta x_{\text{plane}} = \dfrac{w_1 s\,\theta_0}{\sqrt{12}} + \dfrac{w_2 s\,\theta_0}{2} \\ \delta\theta_{\text{plane}, x} = w_2\theta_0 \end{cases} \tag{5}$$

The same has been used for the joint (δy_{plane}, $\delta\theta_{\text{plane}, y}$) distribution in yOz plane. Finally we constructed the incidence points distribution (coordinate distribution) on the detector layer No.2, 3 and 4 (see Fig.2 and Table 1), for an incident 500 MeV/c muon.

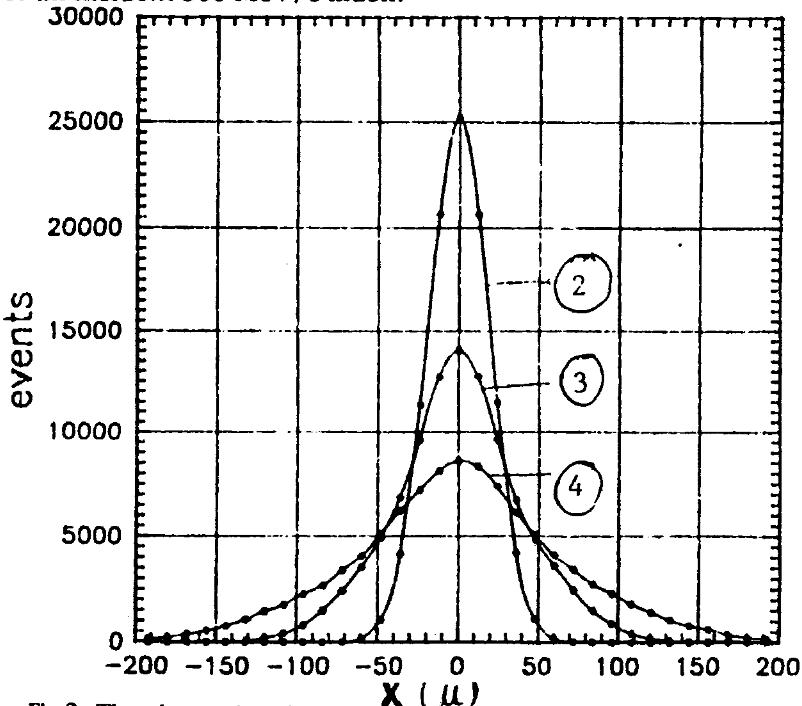

Fig.2. The plan projected x-coordinate point distributions of the scattered 500 MeV/c muons, incident on detector layer No.2, 3 and 4 obtained by Monte-Carlo particle transport simulation

for many applications to use Gaussian approximation for the central 98% of the plan projected angular distribution. The width of this distribution is the root mean square of the scattering angle [3]

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{p\beta c} z_c \sqrt{\frac{s}{X_L}} \left[1 + 0.038 \ln\left(\frac{s}{X_L}\right)\right], \tag{1}$$

where p, βc and z_c are the momentum, velocity and charge number of the incident particle, and X_L is the radiation length of the scattering medium. That is, the plane projected angle $\theta_{\mathrm{plane},\,x}$ or $\theta_{\mathrm{plane},\,y}$ of the deflection angle θ, onto the $x0z$ and $y0z$ planes, where the x and y axes are orthogonal to the $0z$ direction of motion, shows an approximately Gaussian angular distribution

$$\frac{1}{\sqrt{2\pi}\,\theta_0} \exp\left[-\frac{\theta^2_{\mathrm{plane}}}{2\theta_0^2}\right] d\theta_{\mathrm{plane}}. \tag{2}$$

Deflections into $\theta_{\mathrm{plane},\,x}$ and $\theta_{\mathrm{plane},\,y}$ are independent and identically distributed, and $\theta^2_{\mathrm{space}} = \theta^2_{\mathrm{plane},\,x} + \theta^2_{\mathrm{plane},\,y}$.

The angular distribution is translated to a coordinate distribution by particles fly onto every detector layer. The more intersected layers are the larger distribution width is. The coordinate distribution is defined by statistical spread due to MCS, and depends on the number and position of the intersected detector layer elements. It has the same form as angular distribution

$$\frac{1}{\sqrt{2\pi}\,\sigma_{xi}} \exp\left[-\frac{x_i^2}{2\sigma^2_{xi}}\right] dx_i \tag{3}$$

with the mean square deviation (distribution width) as the squares sum of the $(i-1)$ preceding distribution widths projected onto i-th detector layer

$$\sigma^2_{xi} \equiv \langle x_i^2 \rangle = \theta_0^2 \left[(z_i - z_1)^2 + (z_i - z_2)^2 + \ldots + (z_i - z_{i-1})^2\right]. \tag{4}$$

For an oblique incidence ($\theta \neq 0$) the effective path length in the silicon detector is larger, and the same is the position distribution width on the next detector layers. Nevertheless, in this work we will consider only the minimal width (4), as to emphasize the precision limit in particle position measurement with a given silicon tracking system.

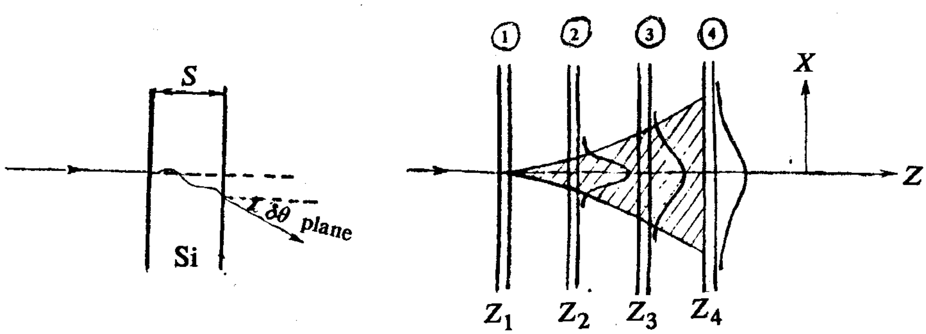

Fig.1. The track particle position uncertainties due to multiple Coulomb scattering. The uncertainty is defined as the coordinate distribution width on every detector layer i and results in the squares sum of all preceding layer scattering contributions projected onto the i-th layer

$$\sigma_i^2 = \theta_0^2\,[(z_i - z_1)^2 + (z_i - z_2)^2 + \ldots + (z_i - z_{i-1})^2]$$

every detector layer. Otherwise, for a good track reconstruction, it is necessary to find the best estimate of the track parameters in a specific particle measurement. This is the reason we studied the particle transport in a silicon tracking system and estimated the multiple Coulomb scattering (MCS) perturbation in particle track measurements.

This study furnishes information on the position resolution we could get from every detector layer, and also its dependence on some physical and geometrical parameters. In the detector system design these parameters could be chosen in an optimal manner, aiming the best track reconstruction possibilities or the best estimate of the track parameters (vertex position, curve radius) directly connected to physical quantities.

The simplified tracking system we used, consists of a 4 layer silicon microstrip detector, 300 μ thick, interspaced by 1.5 cm, and the first detector layer located at 130 cm from the interaction point (see Fig.1).

2. The Track Particle Position Uncertainties Due to Multiple Coulomb Scattering

When a charged particle is traversing the detector elements of a tracking system, it undergoes small deviations of the track, due to MCS. The effect is usually described by the theory of Moliere (see for example [1]). It shows that, by traversing detector's material, thickness s, the particle undergoes successive small-angle deflections, symmetrically distributed about the incident direction. Applying the central limit theorem of statistics to a large number of independent scattering events, the Moliere distribution of the scattering angle can be approximated by a Gaussian one [2]. It is sufficient

z_1 (cm) / p (MeV)	10	30	50	70	90	110	130	150
5500	0.156	1.099	2.905	5.573	9.102	13.494	18.747	24.861
6000	0.153	1.072	2.829	5.425	8.859	13.129	18.240	24.184
6500	0.150	1.051	2.771	5.310	8.668	12.845	17.841	23.658
7000	0.148	1.034	2.723	5.218	8.516	12.619	17.527	23.239
7500	0.147	1.020	2.686	5.144	8.394	12.436	17.273	22.899

The vertex position resolution along x-coordinate (or y-coordinate), due to MCS errors combined with the intrinsic detector coordinate uncertainties, could not be better than the values shown in Table 4. Nevertheless, from Table 4 and Fig.4 we see also that it is possible to have a better vertex position resolution if the detector tracking system is placed to a smaller distance from the interaction point.

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